Ph.D. in Economics, Management & Statistics
University of Catania & University of Messina

Microeconomics

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<u>Instructions</u>: The exam paper consists of two sections. Answer **<u>ONE</u>** question from each section. If you answer more questions from the same section, only the first answer will be marked. You can use standard calculators. Be sure that your mobile phone is switched off and kept in your bag. All the bags should be stored in the place indicated by the invigilator. When you sit in the designed place, be sure that you do not have notes and other unathorized material. If you have such a material, you should store it in your bag. If unathorized material will be found in your possession after the beginning of the exam, you will be asked to leave quietly the room, failing the exam. You have 2.5 hours to complete your exam plus 10 minutes to read the exam paper. If you have any question, raise your hand and ask the invigilator. You **<u>must use</u>** the papers provided to provide your answers. If you need additional papers, ask the invigilator. When handling your exam, remember to write your name both on the exam paper and on those where you wrote your answers. You pass the exam if your average mark is **50 or above**.

Section A

Question 1: .... *100 marks*

Suppose that the preference structure of a consumer is represented by the utility function:

$$U(x_1, x_2) = f(x_1) + x_2$$

with $f'(x) > 0$ and $f''(x) < 0$. These preferences are called quasi-linear because the utility function is linear in good 2. In the first three points of this question you are asked to show some properties of this utility function without assuming a particular functional form for $f(x)$.

(a) [10 Marks] Write down the *Khun-Tucker* first order conditions for the utility maximization problem, given prices $\mathbf{p} = [p_1, p_2]$ and income m.

(b) [20 Marks] Consider now the case of an interior solution. Use the first order conditions to show that good 1 is an ordinary good (negative slope of the marshallian demand). [**Hint**: in the optimum (i.e. substituting back the solution for λ and $\mathbf{x}$) the first order conditions become three identities which are true for all values of the parameters p_1, p_2 and m; therefore, you can differentiate these identities with respect to one of the previous parameter for finding what you are looking for].

(c) [10 Marks] Is good 1 a normal good or an inferior good? Explain and discuss.

(d) [30 Marks] Now assume that:

$$U(x_1, x_2) = \sqrt{(x_1)} + x_2$$

Find the Marshallian demand functions and specify the level of income $\overline{m}$ above which the corner solutions are ruled out.

(e) [15 Marks] In what follows, assume an income level $m > \overline{m}$ to have an interior solution. Find the indirect utility function and check that the indirect utility function is homogeneous (of which degree?) in prices and income; check the validity of the Roy's identity.

(f) [15 Marks] Without solving the expenditure minimization problem, find the Hicksian demand functions and show the symmetry of the cross price effects.

Question 2: .... *100 marks*

You can characterize firm behaviour in two different ways: cost minimization and profit maximization. In the first three points of this question you are asked to show that the two optimization problems are equivalent ways of solving the same problem. Consider the following production function:

$$y = f(x_1, x_2)$$

where $\mathbf{x} = [x_1, x_2]$ is the input vector and $\mathbf{w} = [w_1, w_2]$ is the price vector. Assume that both prices are positive. Assume also diminishing marginal productivity $f_i > 0$ and $f_{ii} < 0 \ \forall \ i = 1, 2$:

(a) [10 Marks] Set up the cost minimization problem and write down the first order conditions with respect to input x_1 and x_2 for an interior solution. This step allows you to obtain the minimum cost function $c(\mathbf{w}, y)$. In the second step, the firm chooses the level of output to maximizes profits. Set up this second optimization problem and write down the first order condition with respect to y. Assume that p is the output price.

(b) [10 Marks] Now, instead of going to the cost-minimization path, set up the one-stage profit-maximization problem and write down the first order conditions with respect to both inputs x_1 and x_2.

(c) [20 Marks] Show that the three first order conditions you obtained in point (a) are equivalent to the two first order conditions from point (b) (**Hint**: you need to use a standard result of the envelope theorem applied to the cost function w.r.t. the output level).

(d) [30 Marks] Continue now under the ffollowing specific assumption:

$$f(x_1, x_2) = x_1^{\alpha}(x_2 - A)^{\alpha}$$

where A and α are positive parameters. Find the corresponding minimum cost function and check its usual properties (increasing in y; increasing and concave in $\mathbf{w}$, homogeneous of degree 1 in $\mathbf{w}$, the Shephard lemma).

(e) [10 Marks] Find the average and the marginal cost functions.

(f) [20 Marks] Suppose that $\alpha = \frac{1}{4}$ and set, for simplicity, $w_1 = w_2 = 1$ and $A = 200$. Find the industry supply function, $Y(p, m)$, for a competitive industry with m identical firms. More, assuming that the industry demand curve is linear, $X(p) = a - bp$, find the equilibrium price and show that the break-even price p^* is equal to 40.

Section B

Question 3: .... *100 marks*

Consider the following game in strategic form:

P2

		W	X	Y	Z
	A	1, 2	2, 1	3, 5	4, 2
	B	5, 3	0, 4	0, 0	5, 7
P1	C	3, 1	1, 2	3, 1	3, 0
	D	2, 3	4, 3	2, 0	2, 0

(a) [20 Marks] Find the equilibrium of the game using the iterated deletion of dominated strategies.

(b) [30 Marks] Explain the Nash equilibrium solution concept and check whether there exists a Nash equilibrium in pure strategy. Explain carefully the steps you take. (**Hint**: Do not write down simply your solution, but explain how you did reach your answer). Discuss the welfare implication of the equilibrium/equlibria you found, if any.

(c) [50 Marks] Find at least one Nash equilibrium in mixed strategies (**Hint**: Besides a pure strategy equilibrium, which is played with probability 1, the game contains 6 mixed strategy equilibria. Some of them assume that a player chooses not to play one or more strategies. The easiest Nash equilibrium in mixed strategy that you can find is the one in which both players randomize over **all** available strategies).

Question 4: .... *100 marks*

Consider a market with three firms 1, 2 and 3. They choose the price, $p_i \in [0, \infty)$ $\forall$ $i = 1, 2$ and 3, that maximizes their profit functions. The quantity is defined as follows:

$$q\left(p_i, p_j, p_k\right) = a - p_i + b_j p_j + b_k p_k \;\; \forall \;\; i \neq j \neq k = 1, 2, 3$$

The profit function takes the following form:

$$\pi_i = q_i\left(p_i, p_j, p_k\right)\left[p_i - c\right]$$

There exists perfect information among firms.

(a) [5 Marks] Write down the maximization problem for each firm.

(b) [25 Marks] Derive the Nash equilibrium for the game above.

(c) [15 Marks] Calculate the profits enjoyed by each firm.

(d) [15 Marks] Now suppose that firm 1 chooses first, then firm 2 observes the choice made by firm 1 and chooses the level of price which maximizes its profit function and, finally, firm 3 makes its decision, after having observed what the other firms chose. The game is, therefore, sequential. Discuss the solution concept that you apply in this case.

(e) [40 Marks] Calculate the quantities that each firm chooses in equilibrium under the scenario in d) and contrast your solutions to those you derived in point b).

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